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DETERMINATION OF THE SHAPE  
OF A FREE PALLOON

Balloons with Superpressure, Subpressure and  
Circumferential Stress; and Capped Balloons

by

Justin H. Smalley

APPLIED SCIENCE DIVISION

Litton Systems, Inc.

2295 Walnut Street

St. Paul, Minnesota 55113

Contract No. AF 19(628)-2783

Project No. 6665

Task No. 666501

Scientific Report No. 3

Report No. 2560

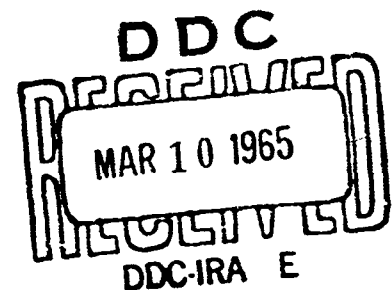
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## NOTICE

The Aerospace Engineering and Research Departments of the Electronics Division of General Mills, Inc. were purchased by Litton Industries, Inc. on 11 September 1963. This acquisition, now known as the Applied Science Division of Litton Systems, Inc., is successor in interest to the Electronics Division of General Mills, Inc. on Contract No. AF 19(628)-2783 per Modification No. 2 to the contract which details the Terms and Conditions of the Novation Agreement entered into as of 11 September 1963 between General Mills, Inc., Litton Systems, Inc., and the United States of America.

## ABSTRACT

This report, the third in a series, continues the presentation of results of the computation of the shape of an axi-symmetric free balloon. Flat-top balloons with superpressure, with subpressure, and balloons with circumferential stress are considered. Circumferential stress is both held constant and varied as a function of meridional stress. Certain limitations on circumferential stress are noted. Shapes, meridional stresses, and circumferential stresses are presented. In addition, similar results are presented for capped balloons with a double-weight cap covering the upper half of the balloon.

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# DETERMINATION OF THE SHAPE OF A FREE BALLOON

Scientific Report No. 3

## I. INTRODUCTION

This report is the third of a series devoted to determining the shape of and stresses in free balloons. Scientific Report No. 1 in this series (Reference 1) pointed out that balloons are being flown today which are outside the range of design parameters provided by the University of Minnesota (Reference 3). Therefore, in Report No. 1, a literature survey was made and the equations defining the shape of a free balloon were derived. In Report No. 2 (Reference 2), the shapes and meridional loads were presented for fully inflated, zero superpressure, zero circumferential stress balloons. Balloons were considered which had all the payload at the bottom, part of the payload at the top, and which had additional lift at the top. Extensive Sigma Tables for the usual flat-top balloon were included.

This report will present shapes and stresses for representative balloons with superpressure, subpressure, and circumferential stress. The case of capped balloons is also presented. Only single bubble balloons with a flat top are considered.

## II. BALLOON TYPES INVESTIGATED

As soon as the restrictions used in Report No. 2 are removed, the possible types and subtypes of balloons increase rapidly. A definitive table of balloon shapes, such as presented in the Sigma Tables of Report No. 2, becomes impractical. With a digital computer, a particular design with all parameters exactly specified without interpolations and extrapolations from charts or tables, can be quickly investigated. In fact, the greatest value of this report may be that it will give the designer a near solution to his problem so that his computer work can be performed expeditiously. The types of balloons investigated follow.

#### A. Superpressure and Subpressure without Circumferential Stress

Balloons investigated in this category were considered to be similar to the polyethylene type. For this reason the maximum superpressure used was  $a/\lambda = 2$ . A maximum subpressure of  $a/\lambda = -2$  more than covers the usual balloon with a truncated duct. Circumferential stress was maintained constant at zero.

#### B. Zero Superpressure with Variable Circumferential Stress

The case of zero superpressure and zero circumferential stress was extensively studied in Report No. 2. Recognizing the fact that seams in polyethylene and Mylar-scrim balloons today are capable of withstanding quite high loads, the effects of nonzero circumferential stress were studied. The circumferential stress ( $t_c$ ) was specified to be some constant fraction of the net meridional stress ( $t_m$ ); that is:  $t_c = t_1 (t_m - t_{m_0})$ , where  $t_{m_0}$  is the initial value of the meridional stress. The reason for this choice is as follows. It is a fundamental result of membrane theory that at a point on the membrane subject to a pressure,  $p$ ,

$$p = \frac{t_c}{R_c} + \frac{t_m}{R_m} \quad (1)$$

where  $R_c$  and  $R_m$  are the circumferential and meridional radii of curvature respectively. From the analysis of Scientific Report No. 1 we have

$$t_m = \frac{1}{r \cos \theta} \left[ \frac{L}{2\pi} + \int_0^s (pr \sin \theta + rw) ds \right].$$

From simple geometric considerations

$$r = R_c \cos \theta.$$

Substituting these two equations in (1) above we have

$$t_c = \frac{pr}{\cos \theta} - \frac{1}{R_m \cos^2 \theta} \left[ \frac{L}{2\pi} + \int_0^s (pr \sin \theta + rw) ds \right]$$

and

$$\lim_{r \rightarrow 0} t_c = \frac{-L}{2\pi R_m \cos^2 \theta_0} .$$

This result indicates that, for  $|\theta_0| < \frac{\pi}{2}$ ,

- 1) If  $t_c$  is to be positive,  $R_m$  must be negative. Such is the case in a rubber balloon.
- 2) If  $R_m$  is positive,  $t_c$  must be a compressive circumferential stress, which, of course, is not possible with balloon materials.
- 3) If  $t_c$  is to be zero, either  $R_m$  must become infinite and/or there is no payload,  $L$ , at the apex.

For  $\theta_0 = \pi/2$ ,  $t_c$  will be zero as  $L = 0$  in this case. It is interesting to note that these results are independent of the pressure in the balloon. For the balloons under consideration in this investigation  $\theta_0$  will be less than  $\pi/2$  and it will be desirable to have some fullness at the apex so the bottom must be conical. That is,  $R_m$  is infinite and  $t_c$  must be zero. Values of  $t_1 = 0.5$ , 1.0, 1.5, and 2.0 were investigated.

### C. Zero Superpressure with Constant Circumferential Stress

It is entirely possible to tailor a balloon gore so that  $R_m < 0$  in the vicinity of the payload. To investigate this case, an estimate of an upper bound on  $t_c$ , when  $r = 0$ , was necessary.

$$\tau = \frac{t_c}{b \lambda^2} = - \frac{L/P}{2\pi (R_m/\lambda) \cos^2 \theta} .$$

For sigma in the range of 0.4 or less, maximum  $\theta_0$  might be 60 degrees. Minimum  $R_m/\lambda$  might be -1 and maximum  $L/P$  will be 1. Therefore, a maximum value for  $\tau$  is on the order of 1. Cases for which  $\Sigma = 0.2$  were studied.

#### D. Capped Balloons

Occasionally free balloons are built with heavier material over some portion near the top. Balloons of this type were investigated to observe how much they differed from constant-material-weight designs. The weight of the heavier material was arbitrarily chosen to be twice that of the material below the cap. The top half of the balloon was again arbitrarily chosen for the capped portion; i. e., the cap extended downward to approximately the point of maximum radius. Sigma values used were 0.1, 0.2, 0.3, 0.4, 0.5 and 0.2, 0.4, 0.6, 0.8, 1.0 for the bottom and top halves respectively. Only zero superpressure and zero circumferential stress were considered.

### III. SHAPE EQUATIONS

The differential equations for the shape of a free balloon have been derived in Scientific Report No. 1 (Reference 1). They are:

$$- \frac{d\theta}{d\sigma} = \left[ \rho(\xi + \alpha) + k \Sigma \rho \rho' - \tau \xi' \right] / \overline{rt}$$

$$\frac{d\overline{rt}}{d\sigma} = k \Sigma \rho \xi' + \tau \rho'$$

where

$$\begin{aligned} \rho' &= \sin \theta \\ \xi' &= \cos \theta. \end{aligned}$$

The initial value of  $\overline{rt}$  is  $(L/P)(2\pi \cos \theta_o)^{-1}$ . The symbols used are defined in Appendix I.

In the previous reports in this series, the circumferential stress,  $\tau$ , has been held zero. In this report it has been defined as:

$$\tau = t_o + t_1 (\tau_m - \tau_{m_o})$$

with  $t_o$  and  $t_1$  permitted to take on values at the investigator's discretion.

The differential form of  $\tau$  could have been included as an additional differential equation. This was not done; instead,  $\tau$  was computed at the end of each increment and then held constant throughout the next increment. This has a negligible effect on the results.

An improved method for computing balloon volume is given in Appendix II.

#### IV. RESULTS

##### A. Superpressure and Subpressure without Circumferential Stress

Shapes and meridional material loads of superpressure and subpressure balloons are shown in Figures 1 and 2.\* In addition to the variation in shape, it is interesting to note that meridional loads are considerably higher in the superpressure types than in the subpressure types. This is due to the large bottom apex angles for superpressure balloons (see Figure 3). Comparison with results for zero superpressure shows that the meridional loads with zero superpressure are also greater than with subpressure. Neglecting the problems of maintaining a relatively large subpressure, considerable savings in weight should be possible with subpressure balloons because a smaller  $\Sigma$  may be used for a given payload. Gore length as a function of  $\Sigma$  is shown in Figure 4.

\* There is considerable difference in scale between Figure 2a and 2b. This is done to present the data more clearly and is not intended to imply a difference in accuracy.

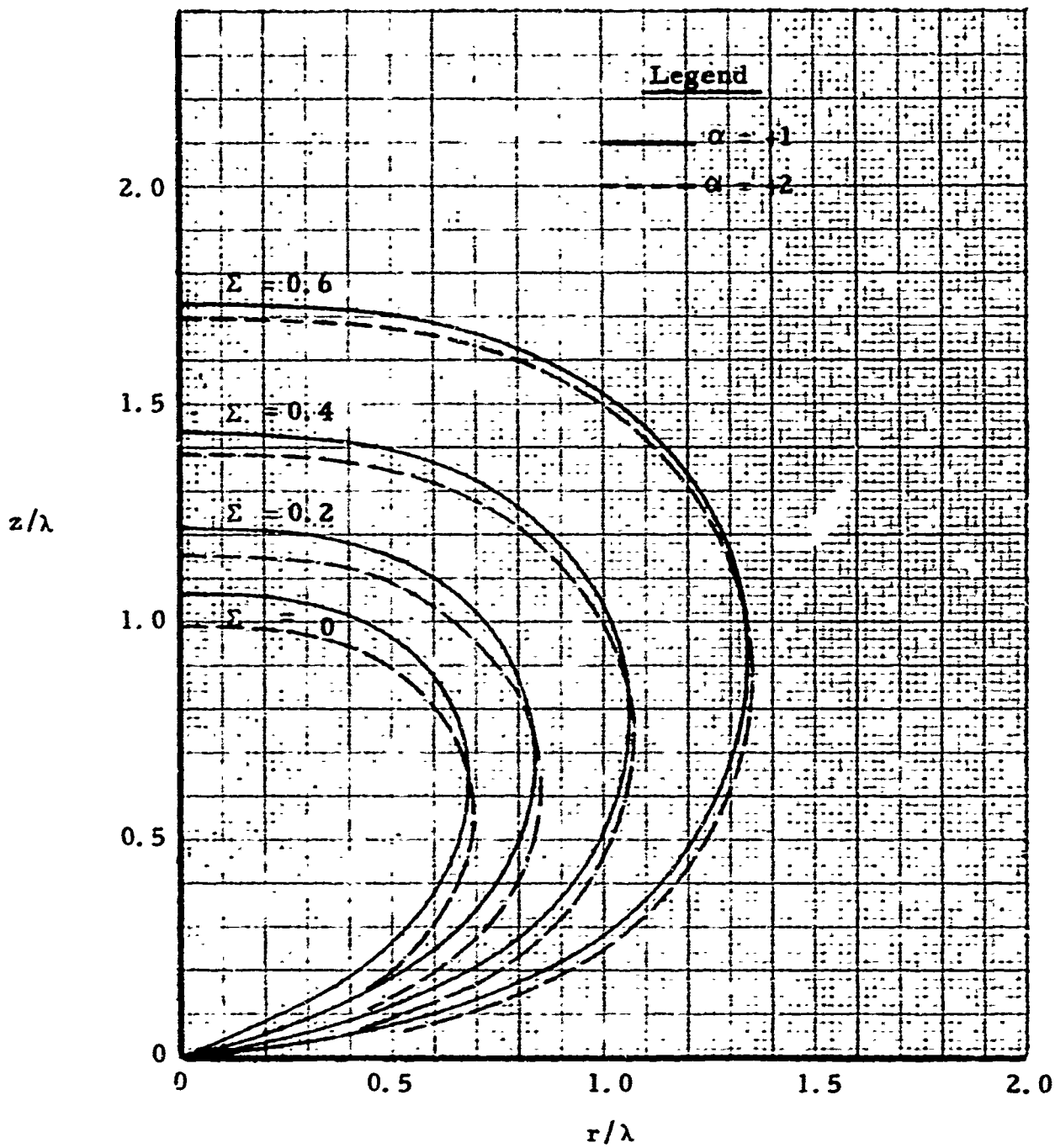


Figure 1. Shapes of balloons with zero circumferential stress

(a) Superpressure types

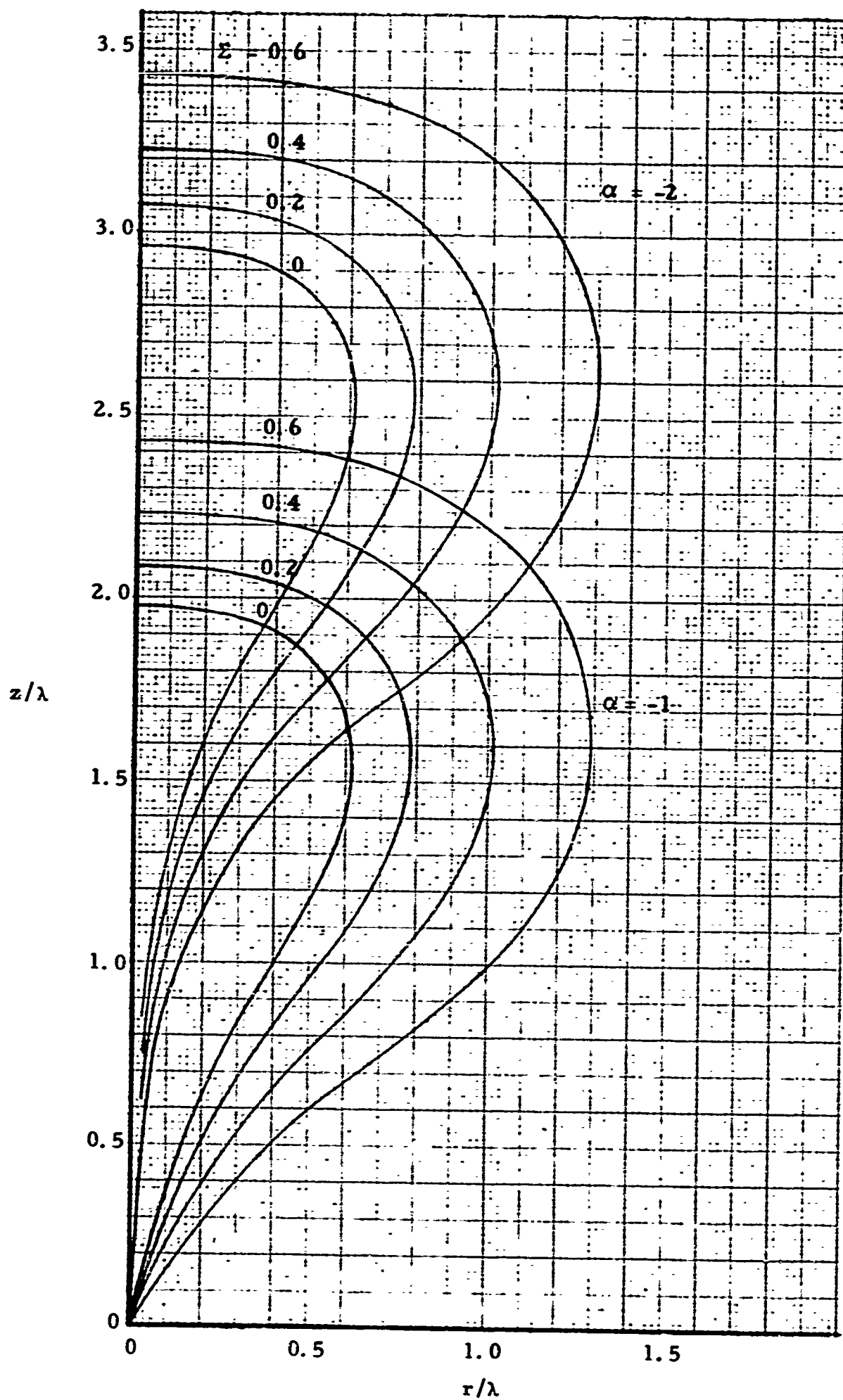


Figure 1. (concluded)  
 (b) Subpressure types

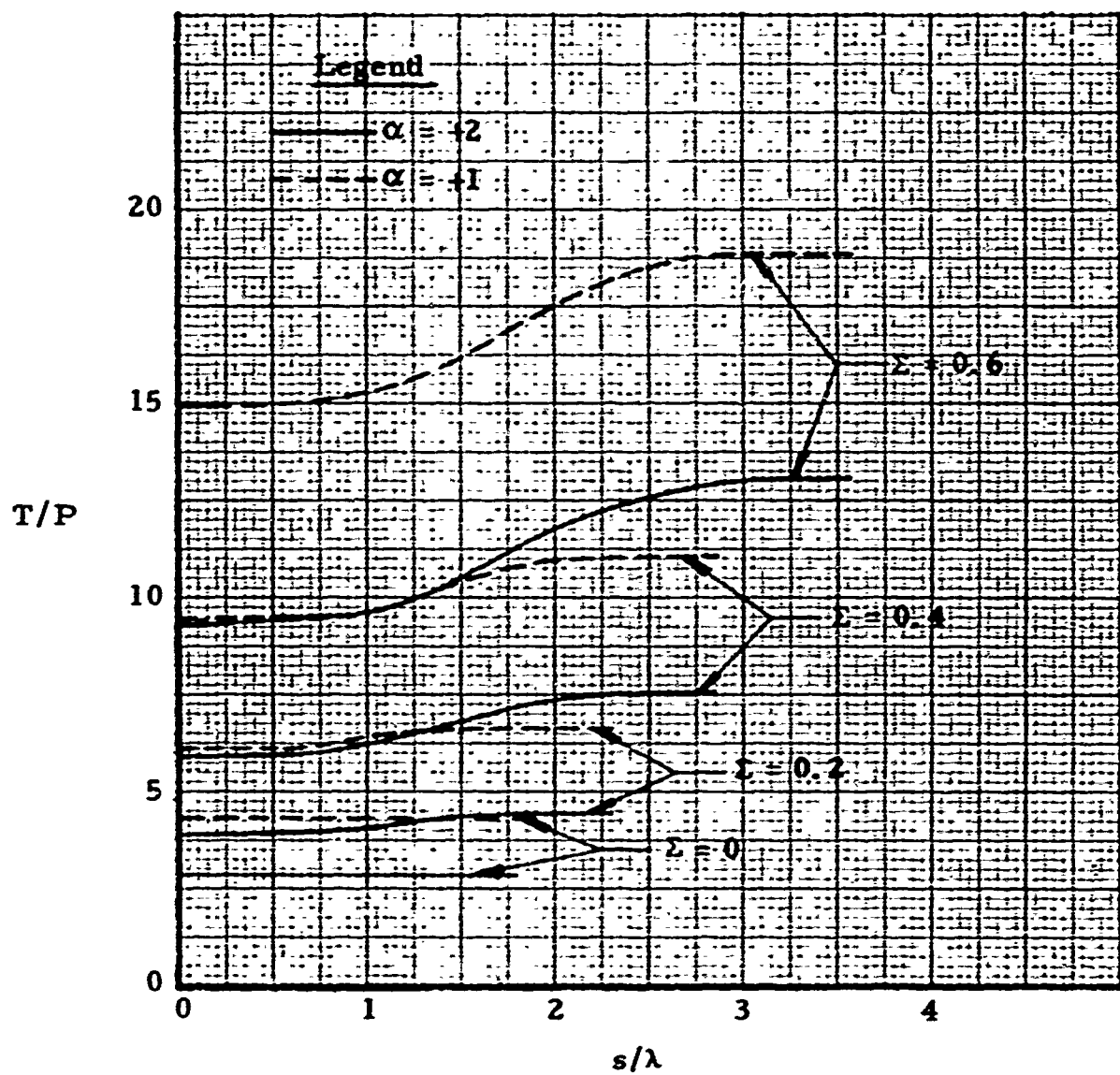


Figure 2. Meridional film load for balloons with zero circumferential stress

(a) Superpressure types



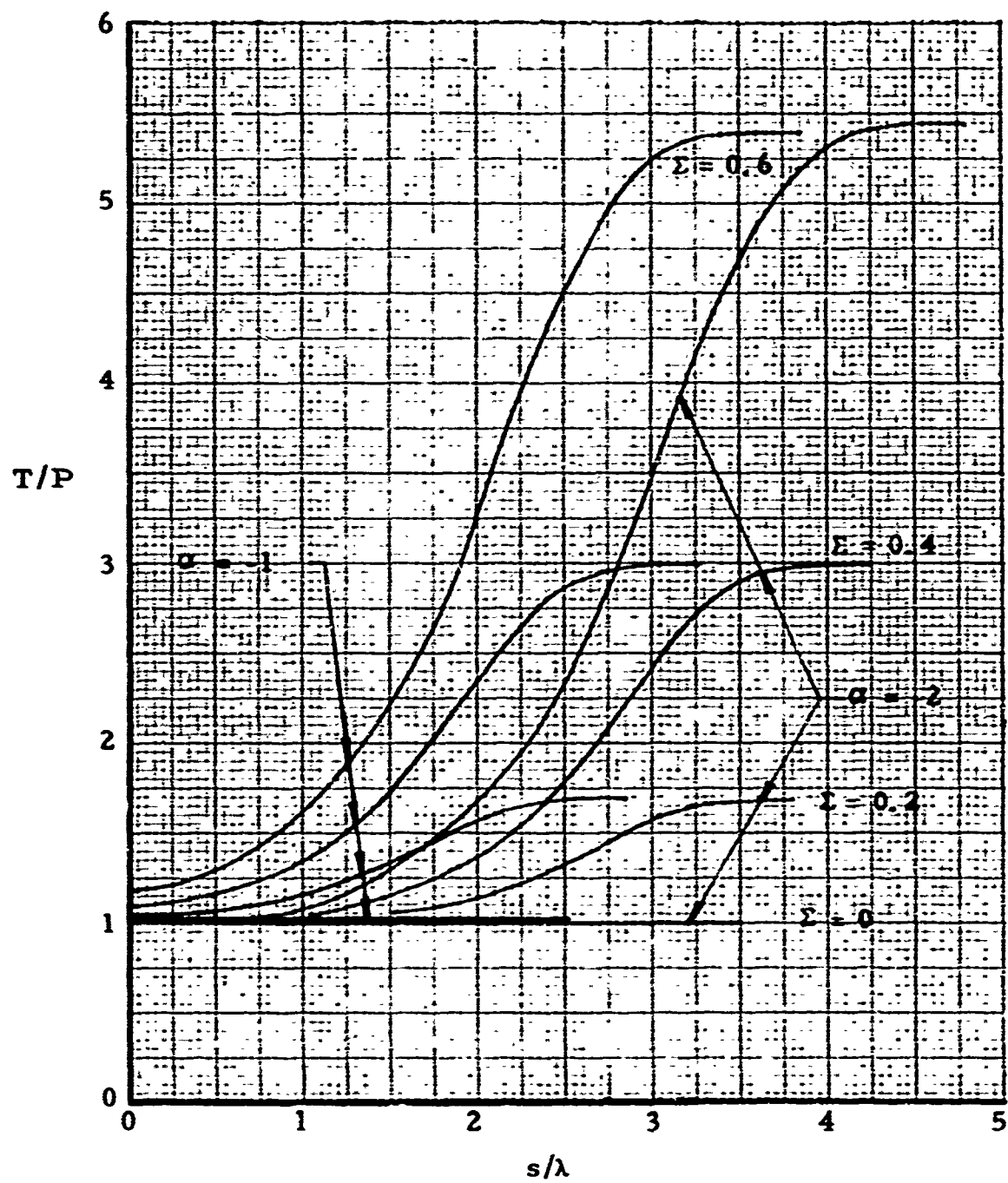


Figure 2. (concluded)

(b) Subpressure types

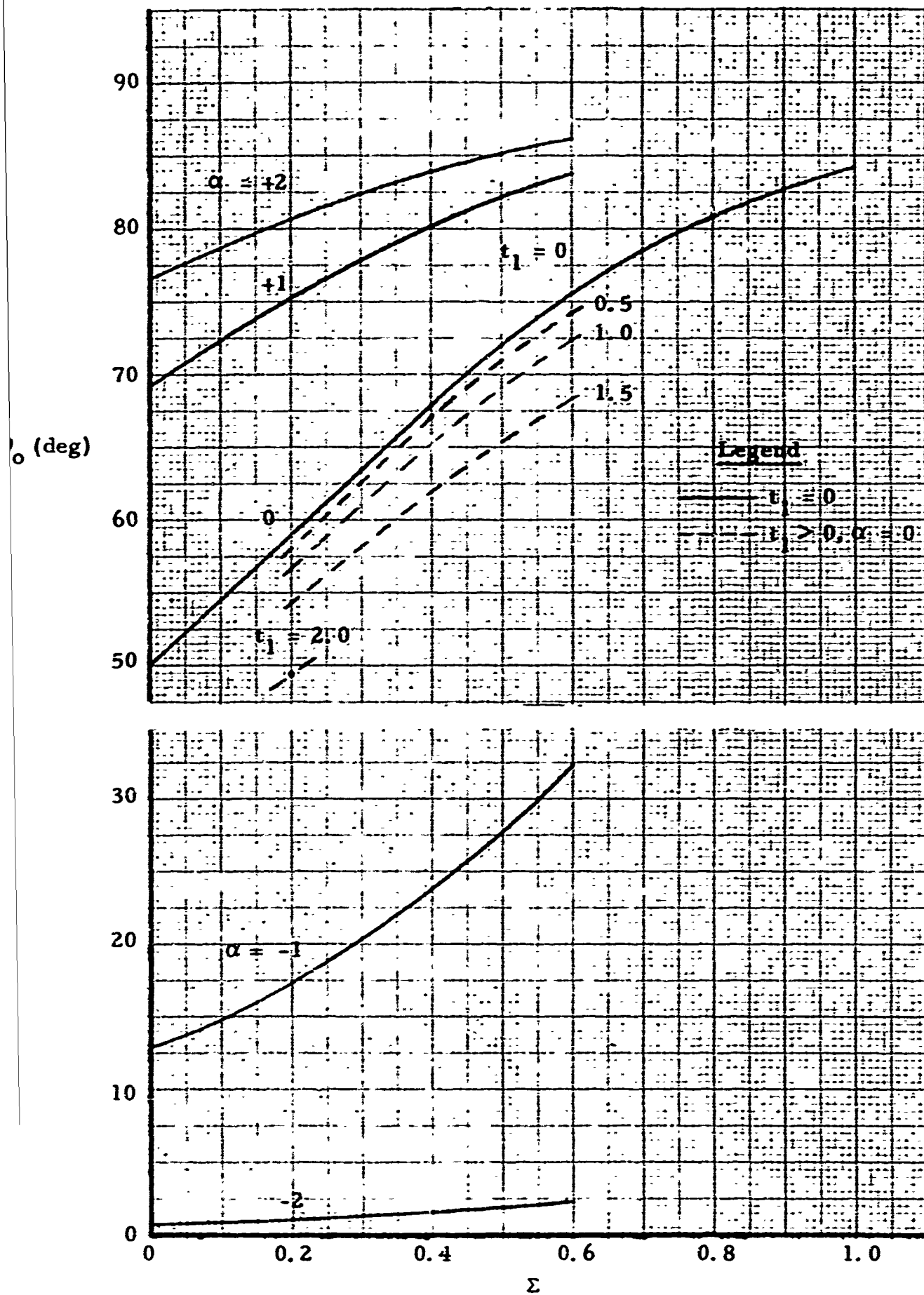


Figure 3. Bottom apex angles for balloons with and without superpressure and circumferential stress

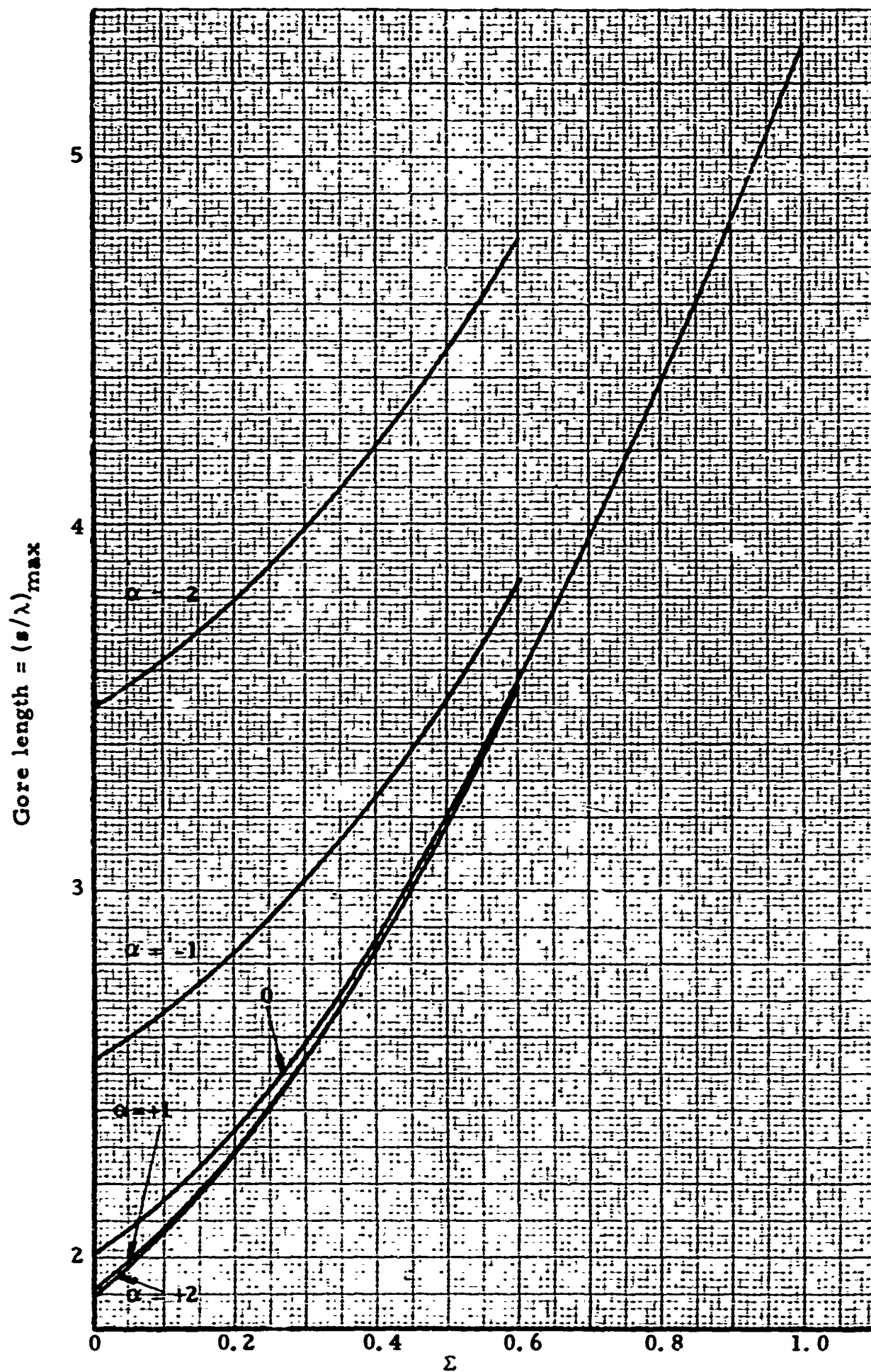


Figure 4. Gore length of superpressure and subpressure balloons (zero circumferential stress)

## B. Zero Superpressure with Variable Circumferential Stress

For balloons with superpressure and a conical bottom it was shown earlier that the circumferential stress must initially be zero. Figure 5 shows such balloon shapes. The bottom apex angle is shown in Figure 3. Meridional loads and corresponding meridional and circumferential stresses are shown in Figure 6. Meridional stress has a minimum near, but a little below, the point of maximum radius as may be expected. As circumferential stress is allowed to take on larger values (i.e.,  $t_1$  is increased), the maximum meridional load is reduced. Also for  $t_1 > 0$  the maximum is not at the top. These two effects are both due to the redistribution of stresses.

After the meridional load passes its maximum and again reaches its initial value, the circumferential stress also reaches its initial value - zero. Beyond this point, the circumferential stress tends to become compressive. If circumferential stress is then held at zero the remainder of the balloon is natural shape. In some of the results of Figure 6a this was the case.

From Figure 6 it can be seen that by proper choice of the constant  $t_1$ , the circumferential stress can be made equal to the meridional stress at some point on the gore. If they are then held equal for the remainder of the gore an approximate sphere top results. In this case meridional stress will not diverge as it usually does.

As seen in Figure 6, meridional stresses diverge at the top of the balloon. Circumferential stresses will usually diverge also. There is one exception. Circumferential stress is calculated as

$$\begin{aligned} t_c &= t_1(t_m - t_{m_0}) \\ &= t_1 \left[ (T/P) - (T/P)_0 \right] \left[ P/2\pi r \right] . \end{aligned}$$

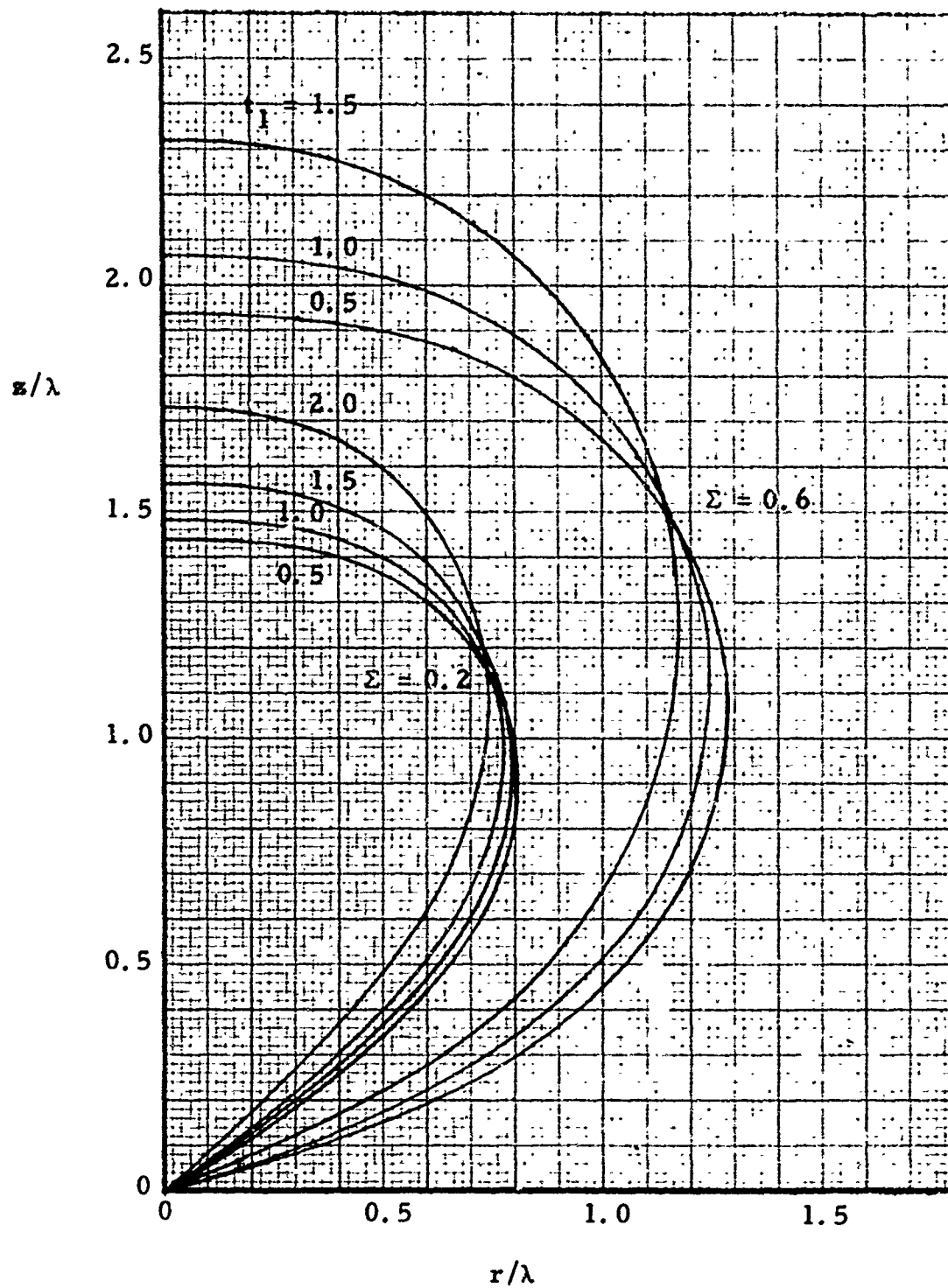


Figure 5. Shapes of balloons with circumferential stress proportional to net meridional stress (Zero superpressure)

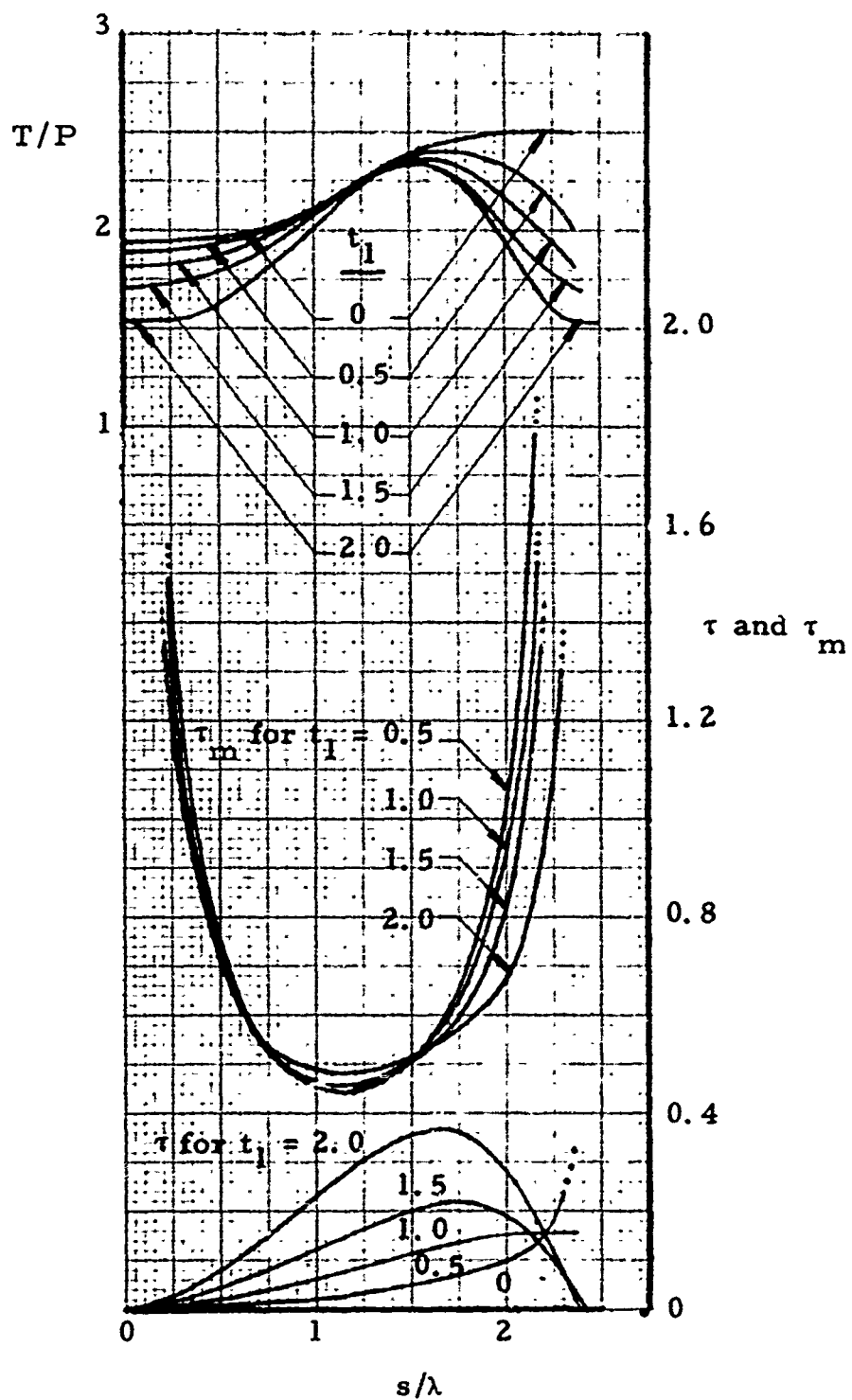


Figure 6. Meridional film loads and circumferential and meridional stresses for balloons with circumferential stress proportional to net meridional stress (Zero superpressure)

(a)  $\Sigma = 0.2$

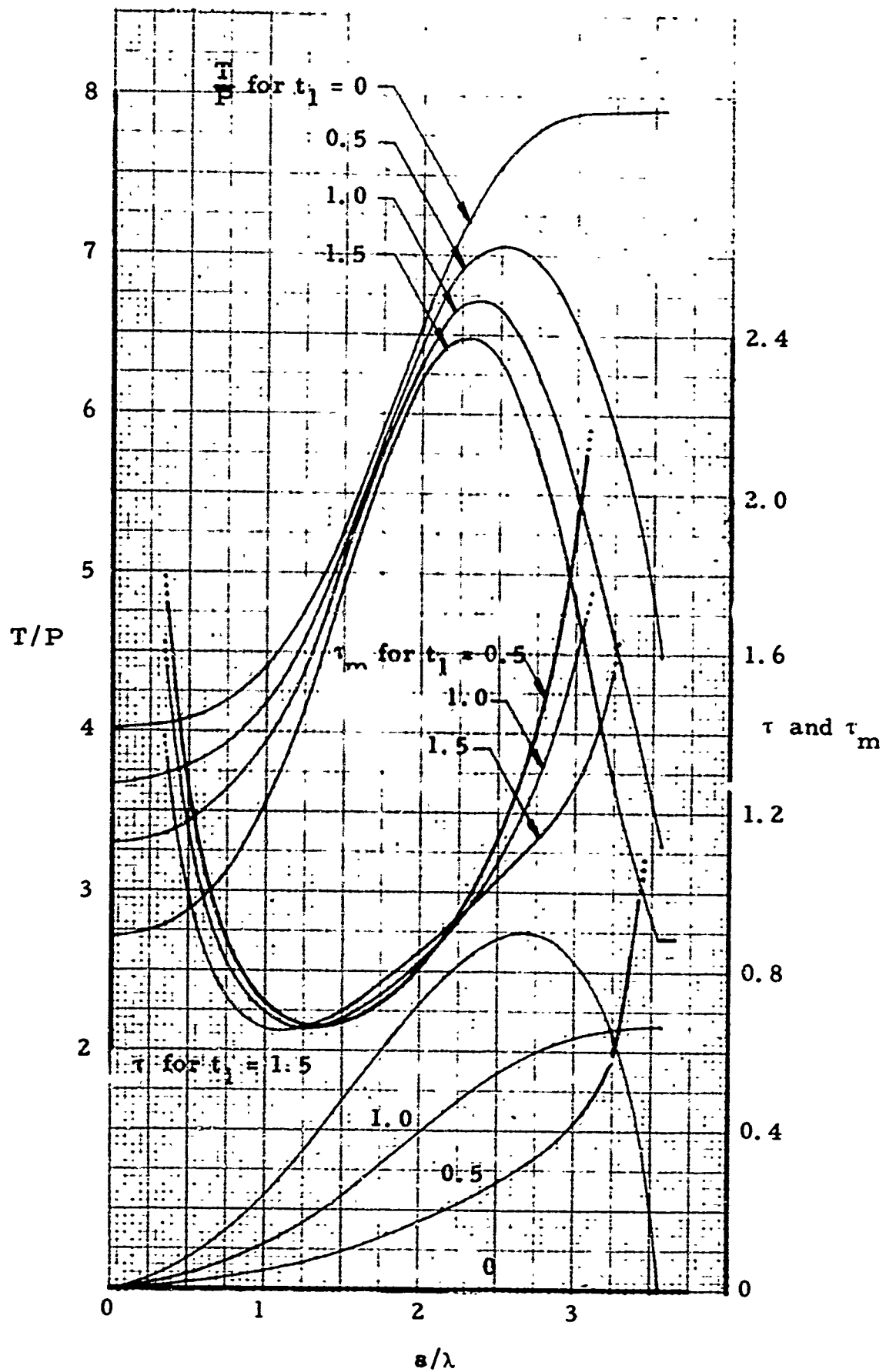


Figure 6. (concluded)

(b)  $\Sigma = 0.6$

In the limit, at the top, as  $r \rightarrow 0$ ,  $t_c \rightarrow \infty$  unless  $T/P = (T/P)_0$ . In this special case

$$\lim_{r \rightarrow 0} t_c = \frac{\lim_{r \rightarrow 0} t_1 P [d(T/P)/ds]}{\lim_{r \rightarrow 0} 2\pi [dr/ds]} = - \frac{t_1 P}{2\pi} \frac{d}{ds} (T/P)_{\text{top}}.$$

It is seen in Figure 6 that the slope of  $(T/P)$  has a finite negative value at the top of the balloon so  $t_c$  will have a finite positive value. No theoretical basis has been developed for it but from the results of Figure 6 it appears that at the top of the balloon  $T/P = (T/P)_0$  when  $t_1 = 1$ . For  $0 < t_1 < 1$ ,  $(T/P)_{\text{top}}$  is greater than  $(T/P)_0$  and  $t_c$  increases without bound. For  $t_1 > 1$ ,  $(T/P)_{\text{top}}$  is less than  $(T/P)_0$  and  $t_c$  attempts to become negative. As this is not possible with balloon materials,  $t_c$  must be held at zero. This prevents any further decrease in  $T/P$ .

As the constant  $t_1$  is increased, the circumferential stress approaches the meridional stress. It was found that when  $\Sigma = 0.6$  and  $t_1 = 2.0$ , it is not possible to have a flat-top balloon. It is obvious from Figure 6b (and from the computer results) that in this case circumferential stress will exceed meridional stress. When  $\Sigma = 0.2$  and  $t_1 = 2.0$  a flat-top balloon is possible, but from Figure 6a it is seen that only a small increase in  $t_1$  would be possible. Addition of superpressure would permit larger values of  $t_1$  but the limitation would still exist that circumferential stress may not substantially exceed meridional stress.

Gore lengths for this type balloon are shown in Figure 7.

### C. Zero Superpressure with Constant Circumferential Stress

In Figures 8, 9, and 10 the balloon shape, meridional load, and bottom apex angle for the case when circumferential stress is constant throughout, is compared with the natural shape results. As predicted in Section II, the lower portion of the balloon has a negative meridional radius of curvature. Due to the presence of circumferential stress the maximum meridional load



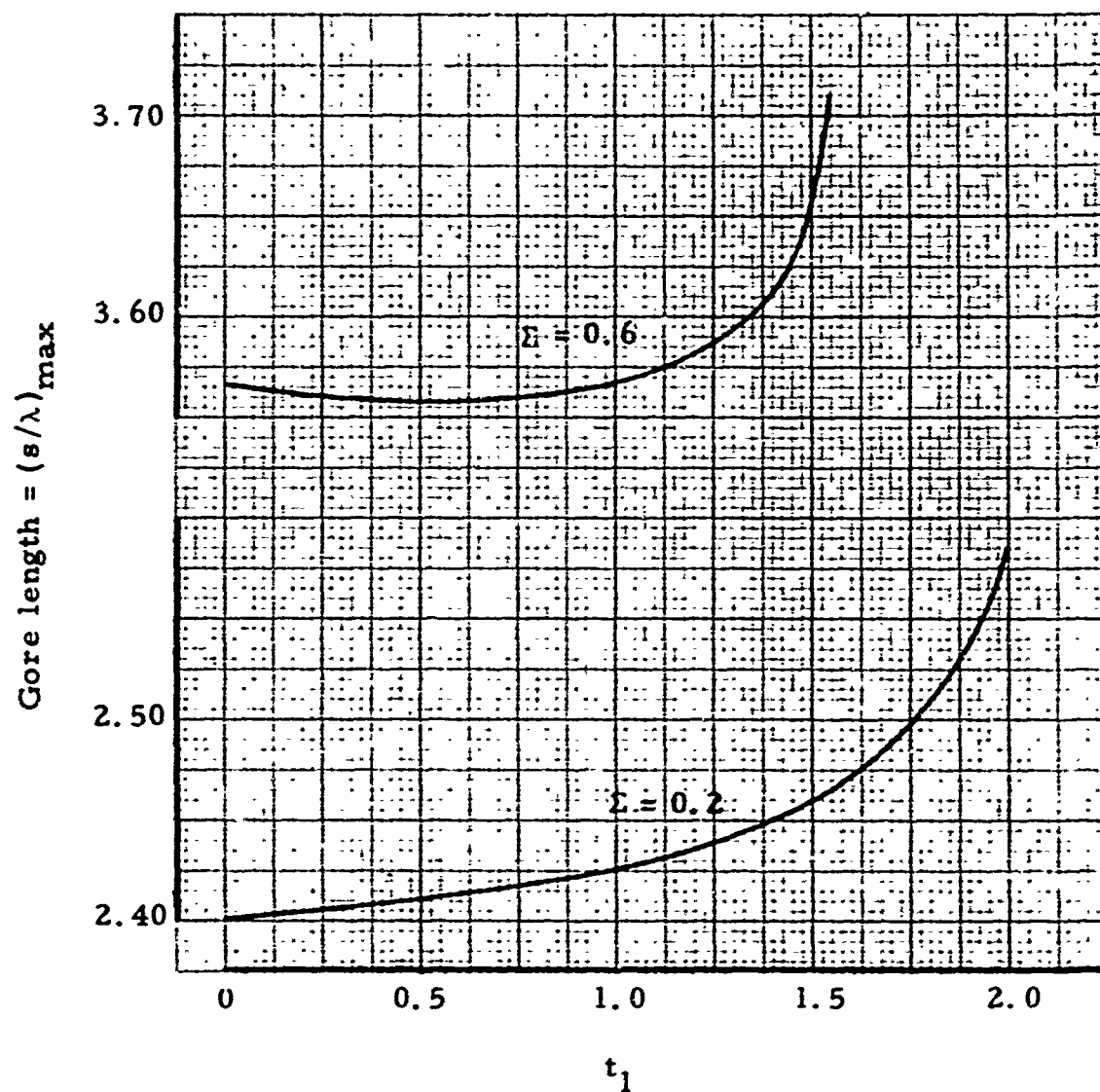


Figure 7. Gore length of balloons with circumferential stress proportional to net meridional stress (Zero superpressure)

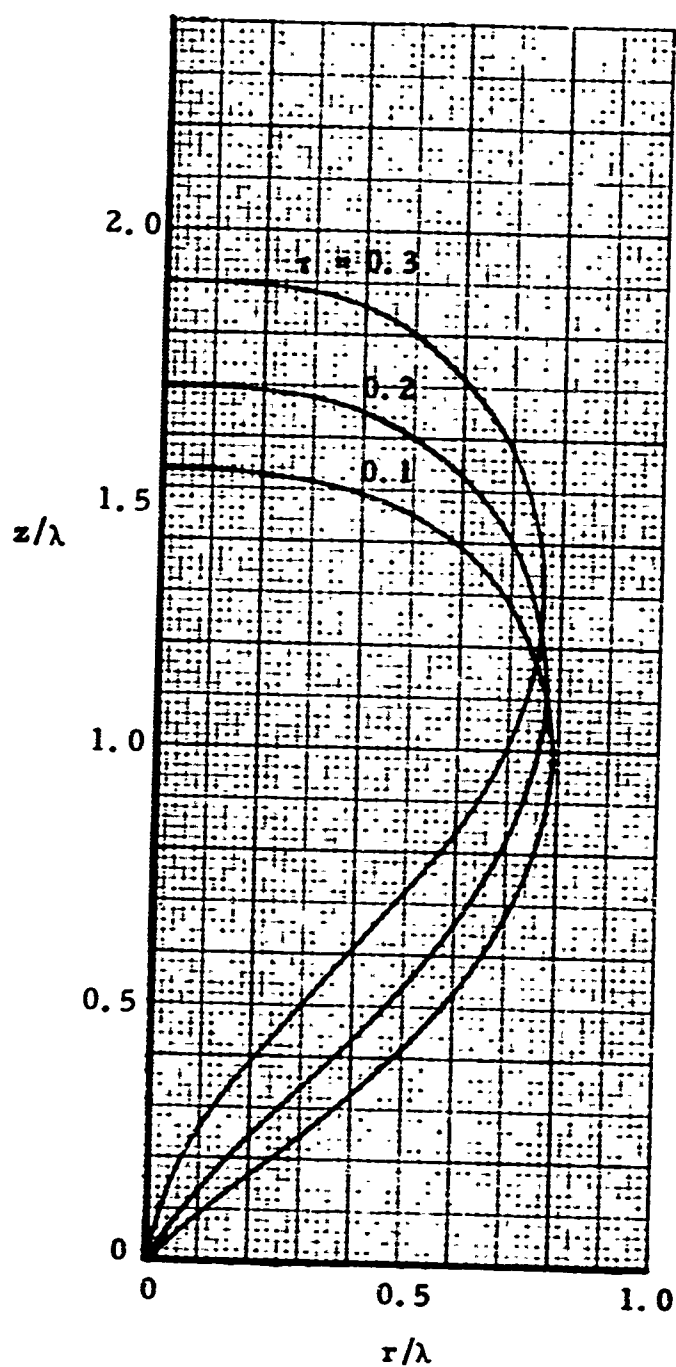


Figure 8. Shapes of balloons with constant circumferential stress  
(Zero superpressure,  $\Sigma = 0.2$ )

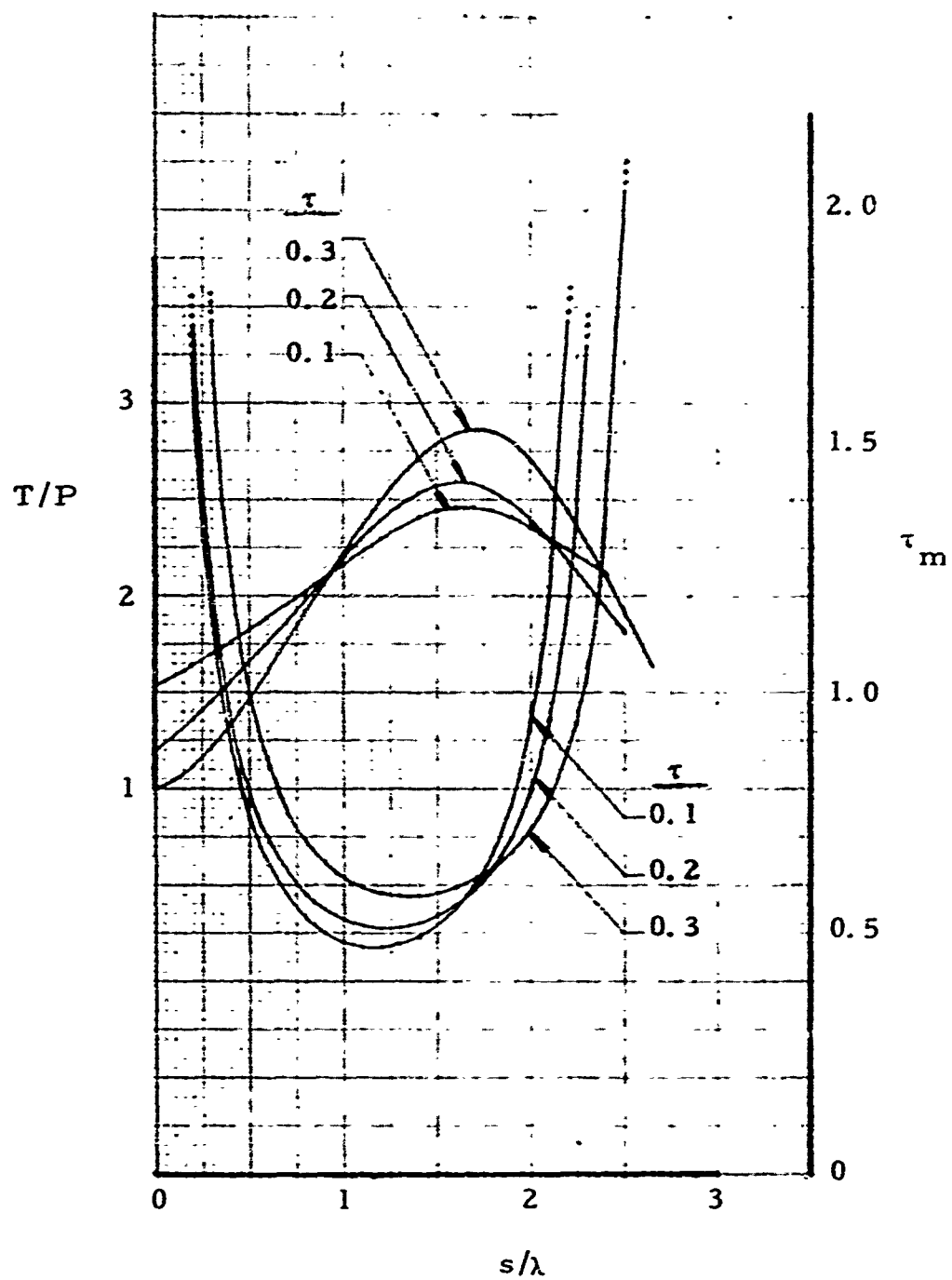


Figure 9. Meridional film loads and stresses for balloons with constant circumferential stress (Zero superpressure,  $\Sigma = 0.2$ )

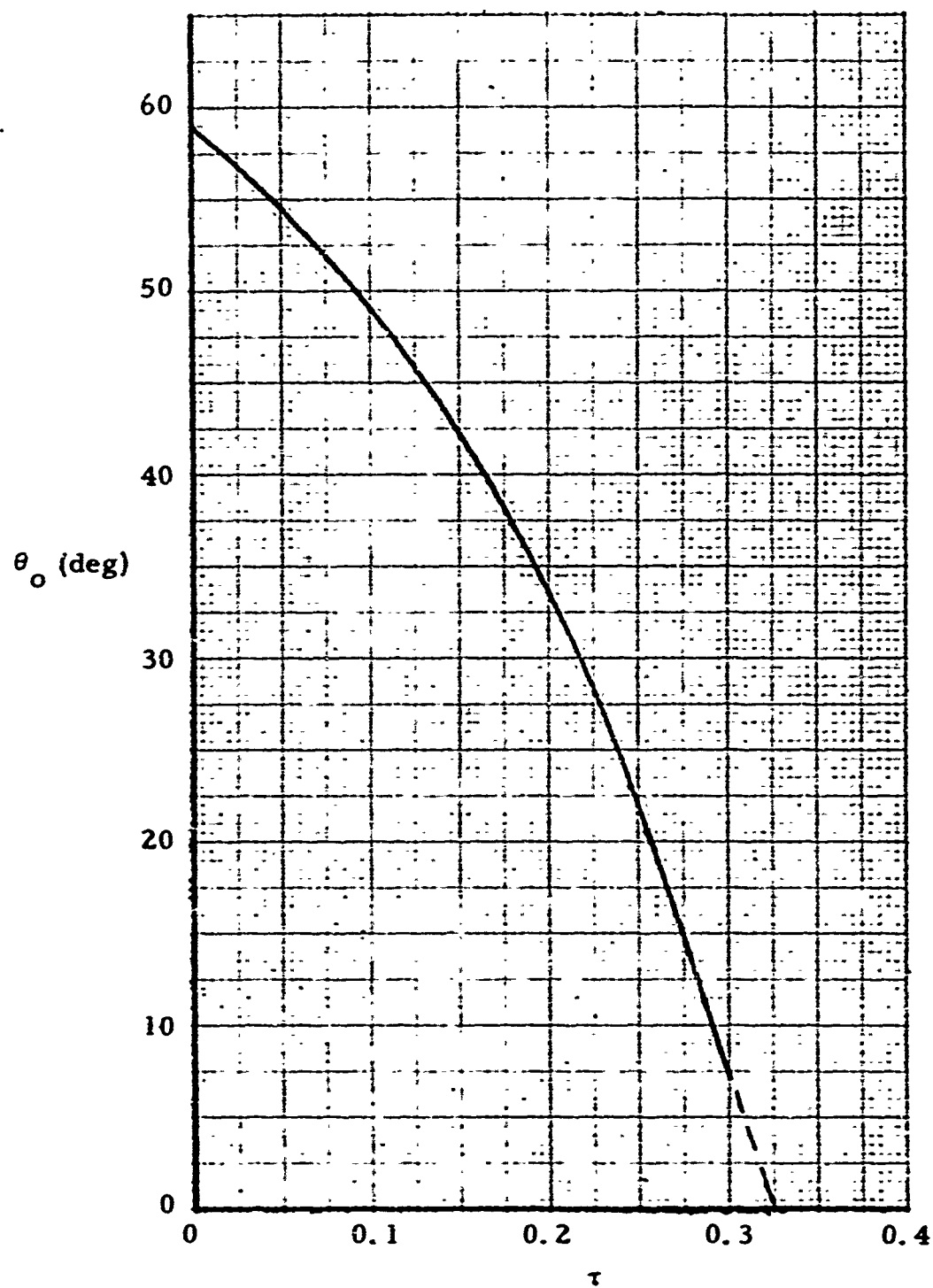


Figure 10. Bottom apex angle for balloons with constant circumferential stress (Zero superpressure,  $\Sigma = 0.2$ )

does not occur at the top. It was found that for  $\Sigma = 0.2$ , a value of  $\tau = 0.4$  would not produce a flat-top balloon. Inspection of Figure 10 shows that the maximum possible value would be about  $\tau = 0.325$ .

#### D. Capped Balloons

The shapes and meridional loads for capped balloons are shown in Figures 11 and 12. Corresponding bottom apex angles and gore lengths are given in Figure 13. The results show that capped balloons are enough different from uncapped balloons, for a given sigma, so that a different gore pattern should be used. In Figure 12, it is seen that there is a discontinuity in the slope of  $T/P$  versus  $s/\lambda$  at the point where there is a change in sigma. It is to be expected that in this case, as the weight of balloon material is doubled, the slope should change by a factor of two.

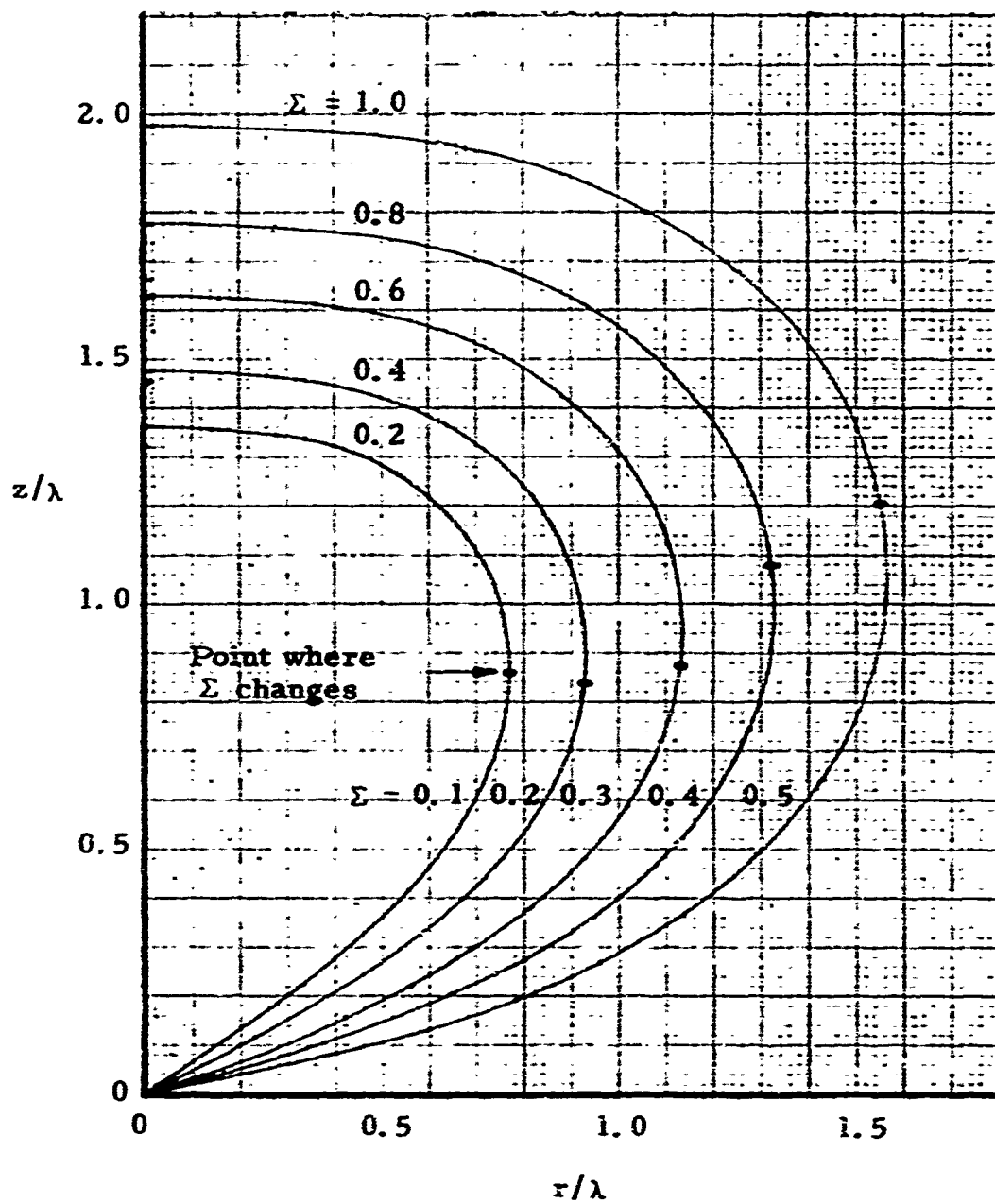


Figure 11. Shapes of capped balloons with a double-weight cap covering the upper half of the balloon  
(Zero superpressure, zero circumferential stress)

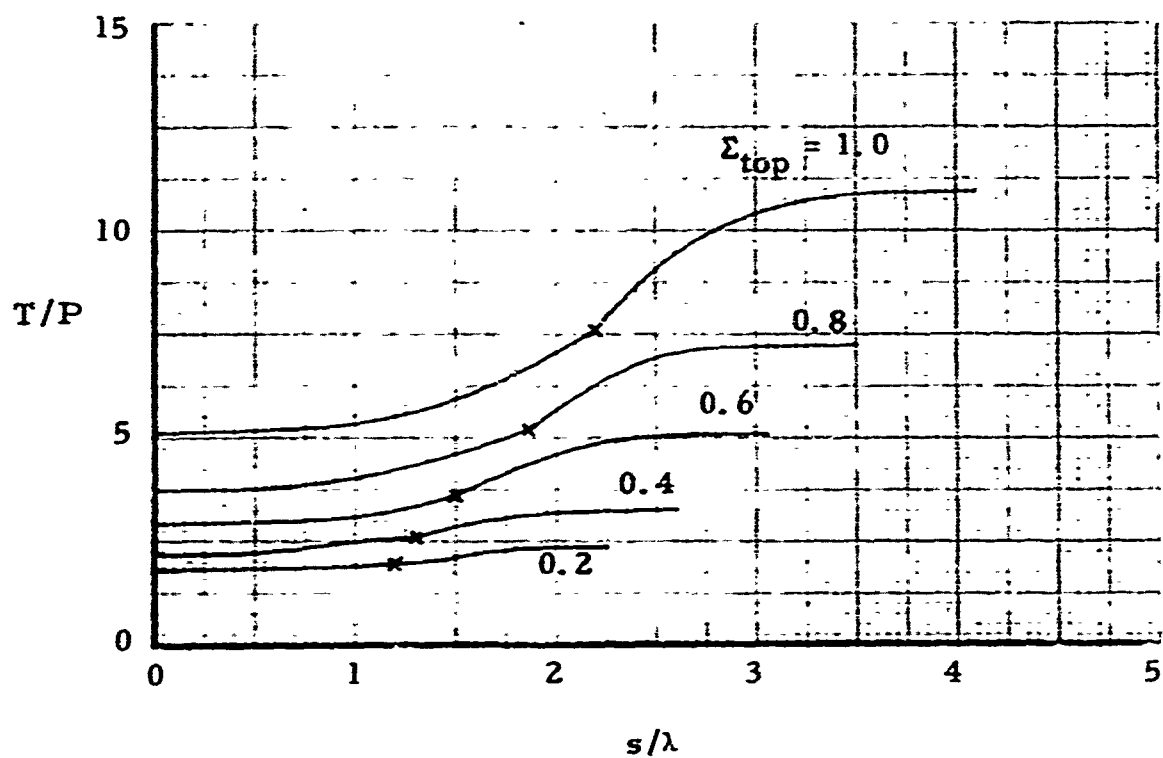


Figure 12. Meridional film loads for capped balloons with a double-weight cap covering the upper half of the balloon  
(Zero superpressure, zero circumferential stress)

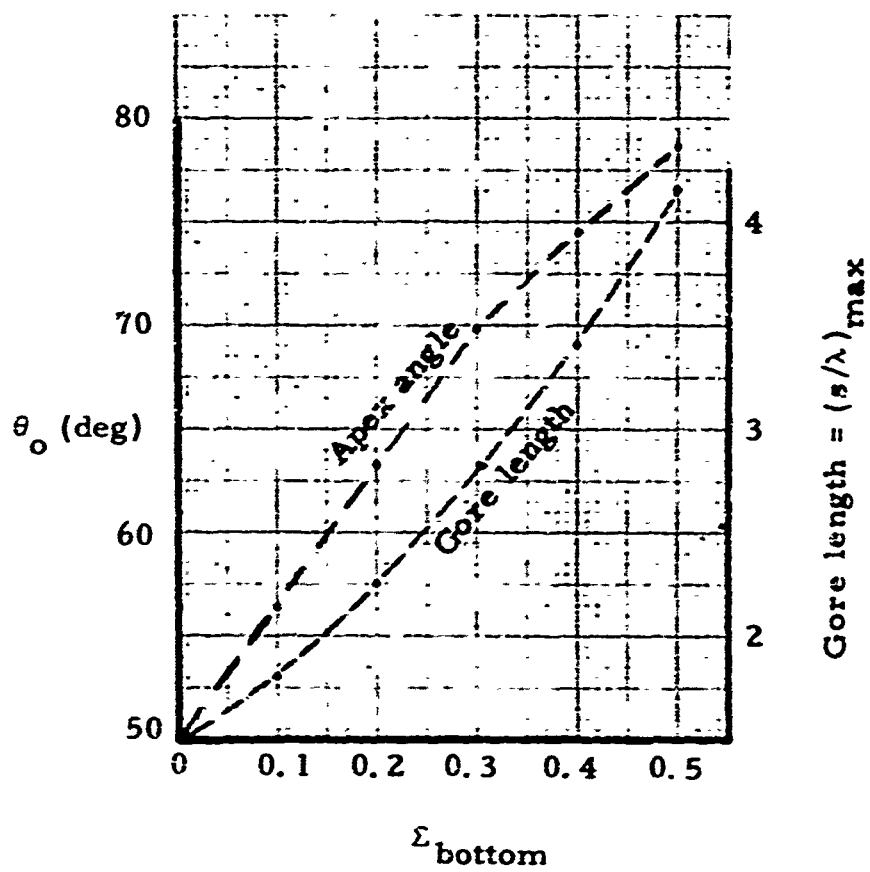


Figure 13. Bottom apex angle and gore length for capped balloons with a double-weight cap covering the upper half of the balloon ( $\Sigma_{\text{top}} = 2 \times \Sigma_{\text{bottom}}$ , zero superpressure, zero circumferential stress)



## V. REFERENCES

- 1) General Mills, Inc. Electronics Division. Report no. 2421. Determination of the shape of a free balloon: Theoretical development by J. H. Smalley. Contract AF 19(628)-2783. Scientific Report No. 1 (Aug. 2, 1963).
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APPENDIX I  
DEFINITION OF SYMBOLS

# APPENDIX I DEFINITION OF SYMBOLS

The symbols used in this series of reports are defined below and illustrated in Figure I-1.

<u>Symbol</u>	<u>Definition</u>	<u>Dimension</u>
a	pressure head at bottom of balloon	length
b	difference in weight densities of air and inflation gas	force per unit volume
k	constant = $(2 \pi)^{-1/3}$	
p	gas pressure across the balloon material	force per unit area
r	radial coordinate of a point on balloon, measured normal to the axis of symmetry	length
$t_c$	circumferential stress in balloon material	force per unit length
$t_m$	meridional stress in balloon material	force per unit length
$t_o, t_1$	constants	
s	gore coordinate of a point on the balloon, measured in the meridional direction from the bottom apex	length
w	unit weight of balloon material	force per unit area
z	height coordinate of a point on balloon, measured parallel to the axis of symmetry from the bottom apex	length
A	area of balloon surface	length squared
B	buoyant force on balloon	force
F	vertical load at top apex of balloon	force

G	gross lift of balloon = $bV$	force
L	payload suspended at bottom apex of balloon	force
P	balloon total payload	force
$R_c$	radius of curvature in the circumferential direction = $r/\cos \theta$	length
$R_m$	radius of curvature in the meridional plane	length
T	total film load = $2\pi r t_m$	force
V	balloon volume	length cubed
W	balloon weight	force
$\overline{rt}$	$= r t_m / P$	
$\alpha$	$= a/\lambda$	
$\xi$	$= z/\lambda$	
$\theta$	angle between tangent to the balloon surface and axis of symmetry, measured in a plane containing the axis of symmetry	
$\lambda$	$= (P/b)^{1/3}$	
$\rho$	$= r/\lambda$	
$\sigma$	$= s/\lambda$	
$\tau$	$= t_c/b\lambda^2$	
$\tau_m$	$= t_m/b\lambda^2$	
$\Sigma$	$= (2\pi)^{1/3} (w/b\lambda)$	

Balloon cross section

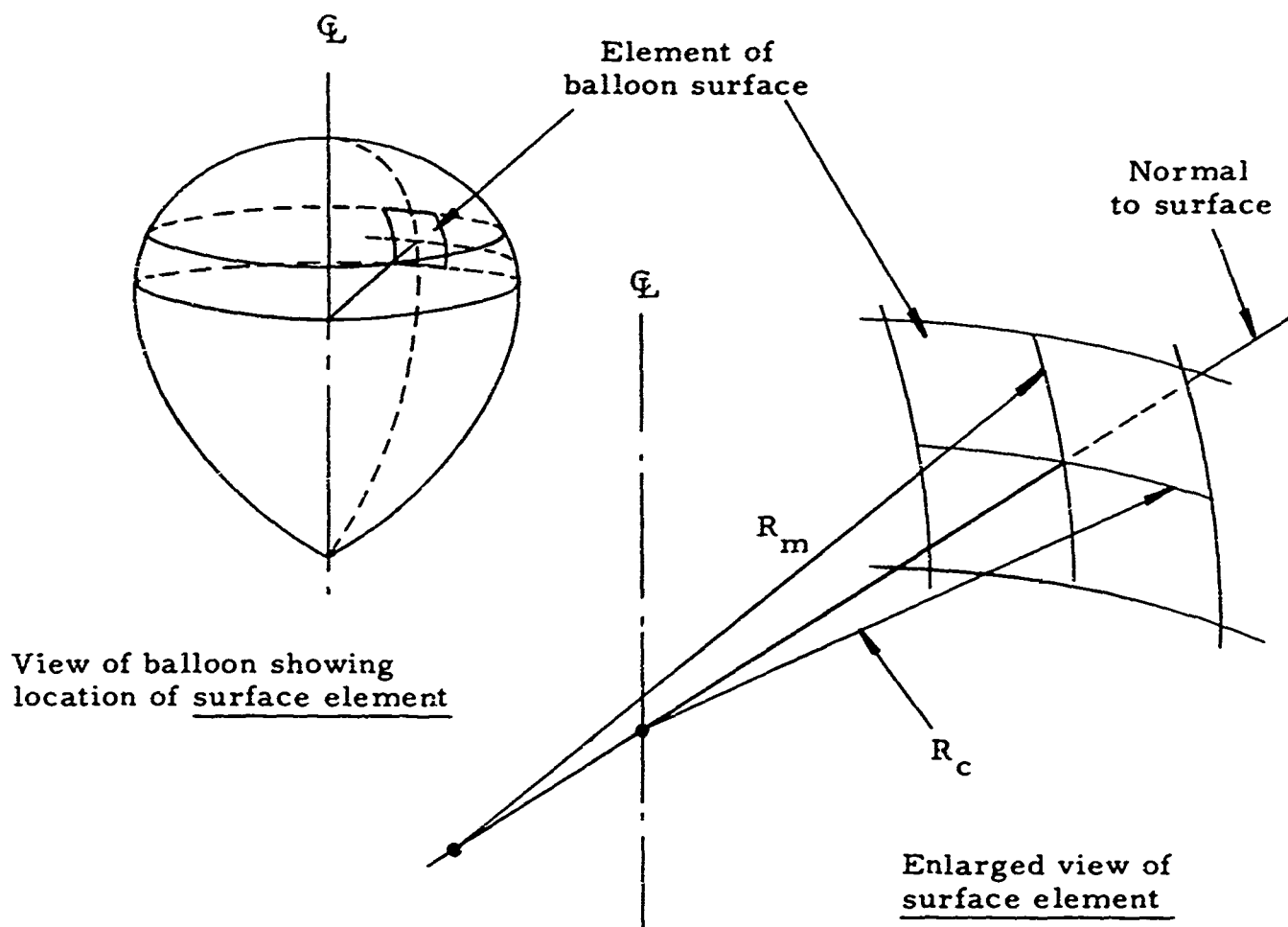
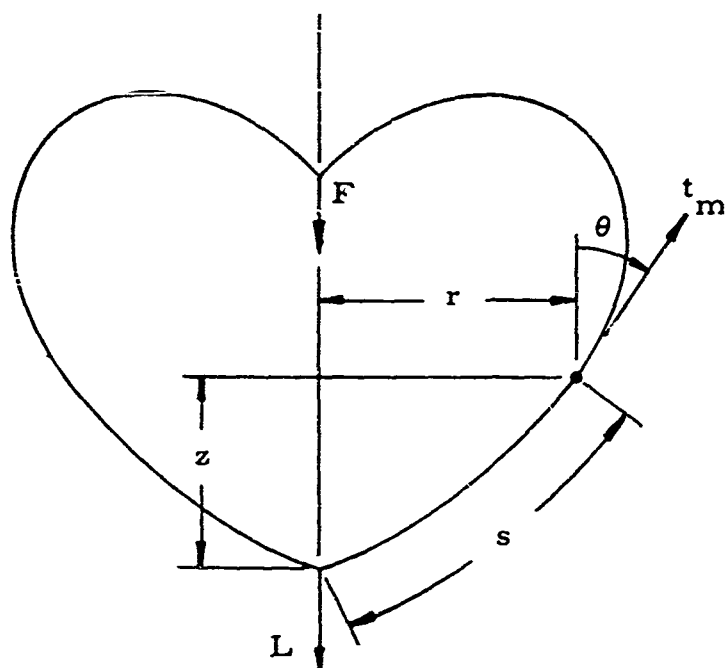


Figure I-1. Pictorial presentation of principal symbols

APPENDIX II  
BALLOON AREA AND VOLUME BY THE GAUSSIAN FORMULA

## APPENDIX II

### BALLOON AREA AND VOLUME BY THE GAUSSIAN FORMULA

In Appendix I of Scientific Report No. 2 (Reference 2) it was shown that if the ordinates and slopes are known at both ends of an increment and if a second degree curve is fitted at these points then

$$y = y_0(1 - k) + y_n(k) - k(1 - k)(y'_n - y'_0)(h/2)$$

where  $(x_0, y_0)$  and  $(x_n, y_n)$  are the initial and final points of the increment,  $y'_0$  and  $y'_n$  are the initial and final slopes  $h$  is the increment width, and

$$k = (x - x_0)/h \quad \text{where } (x_0 \leq x \leq x_n).$$

Using this equation to compute the midpoint of the increment (i. e. ,  $k = 1/2$ ) it was then possible to apply Simpson's Rule over each increment and thereby integrate for surface area and volume of the balloon. Some improvement is possible using Gauss' Formula.

It is stated in Milne (Reference 4) that "These formulas 'Gauss' yield higher accuracy in proportion to the number of points utilized than do other quadrature formulas ...". The formula given there is:

$$\int_0^1 y \, dx = A_1 y(x_1) + A_2 y(x_2) + \dots + A_n y(x_n)$$

where the  $A_i$  and  $x_i$  depend upon the degree of the approximating curve. This formula is exact for equations of degree  $2n-1$ . In general the  $A_i$  and  $x_i$  are irrational which makes them difficult to manipulate, as many places must be carried for maximum accuracy. This objection vanishes when a digital computer is used.

For the problem at hand a fifth-degree approximating curve was chosen. The  $y_i$  were calculated and the  $A_i$  applied as listed in the table below.

$i$	$k_i$	$A_i$
1	0.1127017	5/18
2	1/2	4/9
3	0.8872983	5/18

The  $y_i$  would have been fitted exactly with a third degree curve ( $2n-1$  is odd) but a fifth degree curve was used for integration since the  $y_i$  are squared for determining volume.

Finally, over any particular increment,

$$\frac{\Delta \text{ area}}{2\pi} = (\Delta s)(A_1 y_1 + A_2 y_2 + A_3 y_3)$$

$$\frac{\Delta \text{ volume}}{\pi} = (\Delta z) \left[ A_1 (y_1)^2 + A_2 (y_2)^2 + A_3 (y_3)^2 \right]$$

Computing balloon volume in this way improves the accuracy over that obtainable by use of Simpson's Rule. However, the volume still does not converge as rapidly as the calculation of the balloon shape coordinates or the calculation of the surface area.



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13. ABSTRACT <p>This report, the third in a series, continues the presentation of results of the computation of the shape of an axi-symmetric free balloon. Flat-top balloons with superpressure, with subpressure, and balloons with circumferential stress are considered. Circumferential stress is both held constant and varied as a function of meridional stress. Certain limitations on circumferential stress are noted. Shapes, meridional stresses, and circumferential stresses are presented. In addition, similar results are presented for capped balloons with a double-weight cap covering the upper half of the balloon. (U)</p>		

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14.	KEY WORDS	LINK A		LINK B		LINK C	
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